# B.Sc. $2^{\text {nd }}$ Semester (General) Examination, 2022 <br> Subject: Statistics <br> Paper: GE-II/CC-II <br> (Introductory Probability) 

Time: 2 Hrs
Full Marks: 40

> The figures in the margin indicate full marks. Candidates are required to give their answer in their own words as far as practicable.
> Notations have their usual meaning.

1. Answer any five from the following questions:
$2 \times 5=10$
(a) Give the definition of the sample space.
(b) Write down the classical definition of probability.
(c) What do you mean by a random variable?
(d) Give an example of a continuous random variable.
(e) State weak law of large numbers.
(f) Give the p.m.f of a hypergeometric distribution.
(g) Define the moment generating function of a random variable.
(h) If $\mathrm{P}\left(\mathrm{A}_{1}\right)=0.5, \mathrm{P}\left(\mathrm{A}_{2}\right)=0.3$ and $P\left(\mathrm{~A}_{1} \cap \mathrm{~A}_{2}\right)=0.20$, find $\mathrm{P}\left[\left(\mathrm{A}_{1} \cup \mathrm{~A}_{2}\right)^{\mathrm{C}}\right]$
2. Answer any two from the following questions:
$5 \times 2=10$
(a) State and prove Chebyshev's inequality.
(b) Suppose that the arithmetic mean and the standard deviation of a binomial distribution (with parameters $m$ and $p$ ) are respectively 4 and $\frac{\sqrt{8}}{3}$. Find the value of $m$ and $p$.
(c) A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace?
(d) Derive the moment generating function of geometric distribution.
3. Answer any two from the following questions:
$10 \times 2=20$
(a) (i) State and prove Bayes' theorem.
(i) (ii) If $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ are mutually exclusive events and $P\left(\mathrm{~A}_{1} \cap \mathrm{~A}_{2}\right) \neq 0$, then prove that

$$
\mathrm{P}\left[\mathrm{~A}_{1} \mid \mathrm{A}_{1} \cup \mathrm{~A}_{2}\right]=\frac{P\left(\mathrm{~A}_{1}\right)}{P\left(\mathrm{~A}_{1}\right)+P\left(\mathrm{~A}_{2}\right)}
$$

$$
5+5=10
$$

(b) Suppose that $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ are two independent events. Then show that, (i) $\mathrm{A}_{1}$ and $\mathrm{A}^{\mathrm{c}}{ }_{2}$ are independent (ii) $\mathrm{A}^{\mathrm{c}}{ }_{1}$ and $\mathrm{A}^{\mathrm{c}}{ }_{2}$ are independent.
(c) (i) Show that the mean and variance of a Poisson distribution are equal.
(ii) Suppose $x$ is a Poisson variate with parameter 2. Find $P(X=3), P(X \leq 2)$ and $\mathrm{P}(\mathrm{X}>1)$. [Given $e^{-2}=0.1365$ ]
$5+5=10$
(d) (i) Define a standard normal variable X . Also show that the distribution of X is symmetric. Z
(ii) If Z is a standard normal variate, find the values of $\mathrm{P}[1<\mathrm{Z} \leq 2]$ and $\mathrm{P}[\mathrm{Z} \geq 2]$.

